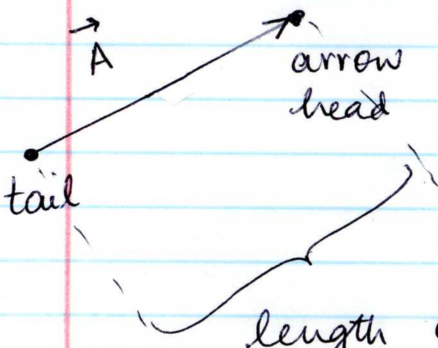
LECTURE 2 VECTORS

Symbol $\vec{A}$ or $\mathbf{A}$ (bold)

A or $|\vec{A}|$ denotes the magnitude or length of $\vec{A}$



length of $\vec{A} = |\vec{A}|$

Scalar : # $\mathbb{Z}$, $\mathbb{R}$, $\mathbb{C}$
↓ integer ↓ real ↘ complex

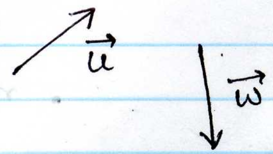
vector : magnitude & direction

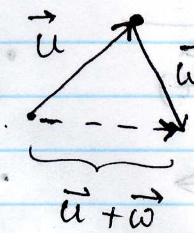
Vector Operations

Addition
Subtraction
Multiplication by scalar
Multiplication (vector by vector)
→ dot product (scalar)
→ cross product (vector)

I. Vector Addition

$\vec{u} + \vec{w} = ?$



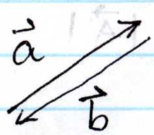


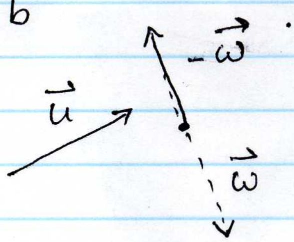
II Vector Substraction

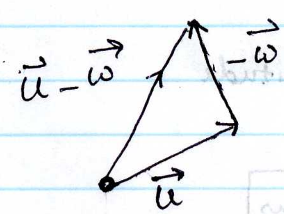
$\vec{u} - \vec{w} = ? \Rightarrow \vec{u} + (-\vec{w})$

"Vector inverse"

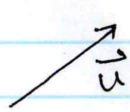
$$\vec{a} + \vec{b} = 0 \Rightarrow \vec{a} = -\vec{b}$$



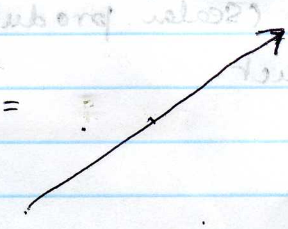




III Multiplication by a scalar



$2\vec{u} =$

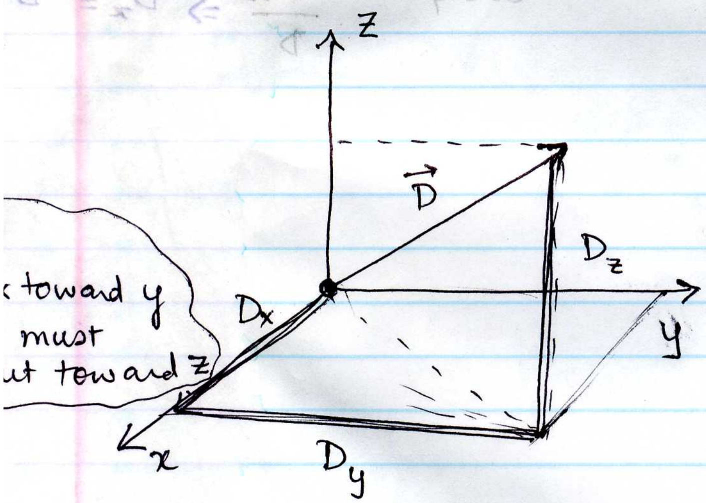


increase magnitude by this factor.

$-0.5\vec{u}$

$\boxed{\text{Components}}$ of a vector

* How many dimensions does the vector live in?
* Set up a (right handed) system of mutually perpendicular grid axes with an origin O.
* Place the origin of the vector at O.
* find the projections of the vector along the axes of the grid.



components

$$\vec{D} = D_x \hat{x} + D_y \hat{y} + D_z \hat{z}$$

scalar (magnitude along x; can be < 0)

unit vector along $\hat{x}$

$$\vec{u} = \begin{bmatrix} u_x \\ u_y \\ u_z \end{bmatrix} \qquad \vec{u} \pm \vec{w} = \begin{bmatrix} u_x \\ u_y \\ u_z \end{bmatrix} \pm \begin{bmatrix} w_x \\ w_y \\ w_z \end{bmatrix} = \begin{bmatrix} u_x \pm w \\ u_y \pm w \\ u_z \pm w \end{bmatrix}$$

Magnitude $\vec{u}$ $\doteq u = \sqrt{u_x^2 + u_y^2 + u_z^2}$
(length)

always ≥ 0

Multiplication by scalar $\quad b\vec{u} = b\begin{bmatrix} u_x \\ u_y \\ u_z \end{bmatrix} = \begin{bmatrix} bu_x \\ bu_y \\ bu_z \end{bmatrix}$