LECTURE 3 COLLISIONS

MOMENTUM CONSERVATION

One dimension: Consider perfectly inelastic collisions

⇓

2 objects collide and stick together

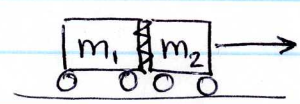
I

Initial

v_1^{in} $v_2^{in} = 0$

[v_1^{in} is known]

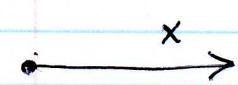
Final



v_f [find v_f ?]

Momentum Conservation ⇒ $\vec{P}_{tot}^{\,in} = \vec{P}_{tot}^{\,f}$

$$\vec{P}_1^{\,in} + \vec{P}_2^{\,in} = \vec{P}_{tot}^{\,f}$$



$$m_1 v_1^{in} + m_2 \cdot 0 = (m_1 + m_2)\, v_f$$

$$\Rightarrow \boxed{v_f = \frac{m_1 v_1^{in}}{m_1 + m_2}}$$

Special cases:

a) $m_1 = m_2 = m$ Equal masses

$$v_f = \frac{m}{2m} v_1^{in} = \frac{v_1^{in}}{2}$$

Final speed of the stuck masses is half the original speed of the moving object.

Does this make sense?

b) $m_2 \gg m_1$ The stationary ~~mass~~ object is much much heavier than the moving object
e.g. a fly hitting a car
or a car hitting a brick w

$$v_f = \frac{m_1 v_1^{in}}{m_1 + m_2} \simeq \frac{m_1}{m_2} v_1^{in}$$

(because $m_1 + m_2 \approx m_2$)

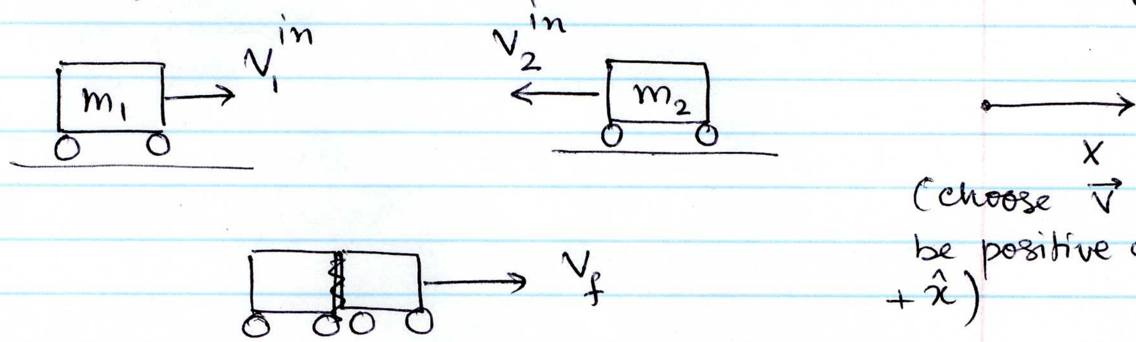
as $m_2 \to \infty$ $v_f \to 0$

Does this make sense?

II. Both objects are moving with known velocities $\vec{v}_1^{\,in}$ and $\vec{v}_2^{\,in}$; they collide and get stuck. Find the final velocity

Initial

Final



Momentum Conservation $\Rightarrow$ $\vec{P}^{\,in}_{tot} = \vec{P}^{\,f}_{tot}$

$$\vec{p}_1^{\,in} + \vec{p}_2^{\,in} = \vec{p}^{\,f}_{tot}$$

$$m_1 v_1^{in} - m_2 v_2^{in} = (m_1 + m_2)\, v_f$$

$$\boxed{v_f = \frac{m_1 v_1^{in} - m_2 v_2^{in}}{m_1 + m_2}}$$

Special cases

$$m_1 v_1^{in} - m_2 v_2^{in} = (m_1 + m_2) v_f$$

$$\boxed{v_f = \frac{m_1 v_1^{in} - m_2 v_2^{in}}{m_1 + m_2}}$$

Special cases

(a) Both objects are the same mass $m_1 = m_2 = m$ moving with equal and opposite velocities $v_1^{in} = v_2^{in} = v_0$

$\Rightarrow \quad v_f = 0$

(b) $m_1 \neq m_2$ $\quad v_1^{in} = v_2^{in} = v_0$

Both objects move with equal and opposite velocities but have different masses.

$$v_f = \frac{m_1 v_0 - m_2 v_0}{m_1 + m_2}$$

$$v_f = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) v_0$$

$$\boxed{v_f = \frac{\Delta m}{M} v_0}$$

Where $\Delta m = m_1 - m_2$ difference in mass

$M = m_1 + m_2$ = total mass

For the same total mass M as Δm is increased the final velocity ~~is~~ increases

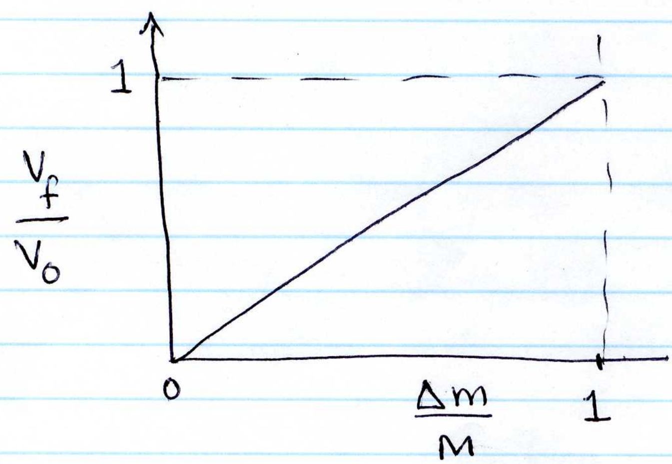
$(m_1 + m_2)$

$$V_f = \frac{\Delta m}{M} V_0$$

For the same total mass M as Δm is increased the final velocity ~~is~~ increases

Where $\Delta m = m_1 - m_2$
difference in mass

$M = m_1 + m_2$
= total mass



Try to put the graph into words.

We are given some initial velocity v_0 with which v_1 and v_2 move

We are given their total mass $M = m_1 + m_2$

But $\Delta m = m_1 - m_2$ can vary.

The graph shows that when $m_1 = m_2$ the final velocity of the stuck masses $= 0$

As one of the masses gets heavier the combined mass has a net velocity.

Finally, when $m_1 \to M$ as the mass of one of the objects tends to the total mas and the other object is essentially tending t zero then the final velocity of the combine mass equals that of the mass M.

In other words this mass M does not see the very very light mass.