Lecture 8 9/4/2012

ENERGY CONSERVATION

(1) Consider simple case of object moving with velocity $\vec{V}$

The momentum of object $\vec{p} = m\vec{V}$

mass velocity

units $[p] = [kg][m/s] = kg\ m/s$

Because of the motion, the object has KINETIC ENERGY

$$k = \underbrace{\frac{1}{2} m V^2}_{\text{scalar}} = \frac{1}{2} m \vec{V} \cdot \vec{V}$$

$$= \frac{1}{2m} m^2 V^2 = \frac{\vec{p} \cdot \vec{p}}{2m} = \frac{p^2}{2m}$$

Kinetic energy is a scalar with units

$$[k] = kg \frac{m^2}{s^2} \equiv \text{Joule}$$

It depends on the mass and the speed.

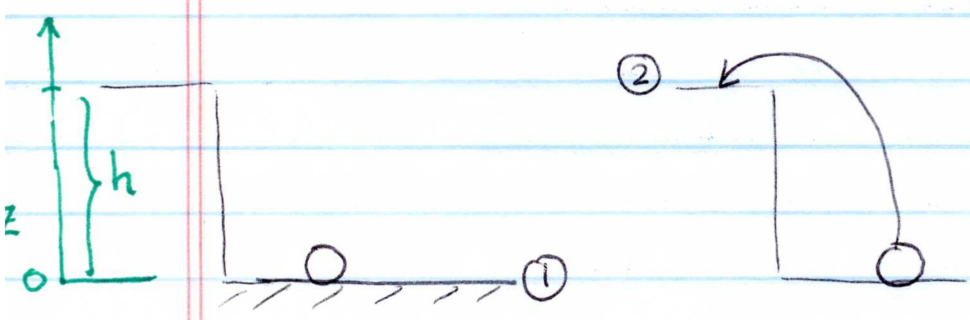
(2) Potential Energy :

arises because of interactions

e.g. gravitational interaction.

$$V(z) = mgz + C$$

$\uparrow$ constant.



$$\boxed{\begin{array}{c} g = 9.8\ m/s^2 \\ \text{acceleration due} \\ \text{to gravity} \end{array}}$$

object is on the ground

Set up the axis and define $z = 0$ on the ground

$$\Rightarrow V_1 = mg \cdot 0 + C$$

$\uparrow$

potential energy of ball when it is on the ground

Object is on the table a height h abo

the ground.

$$V_2 = mgh + C$$

$\uparrow$

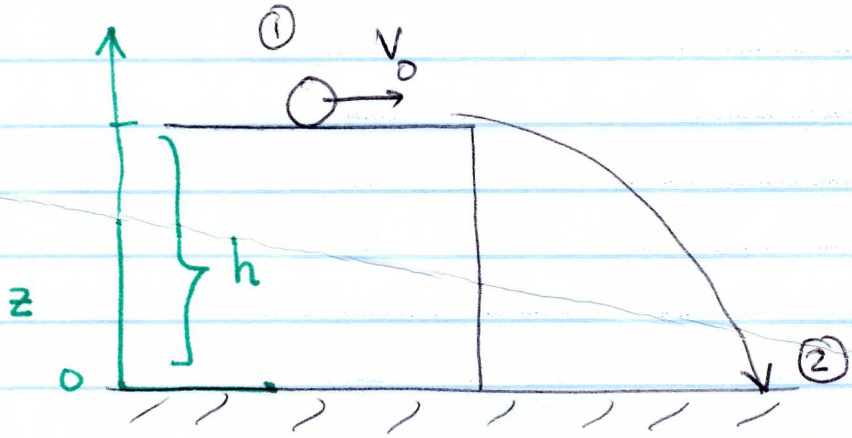
potential energy of ba

when it is on the table

The difference in potential energy

$$\boxed{\Delta V = V_2 - V_1 = mgh}$$

(3) Object falls off the table



Initial velocity of object $= \vec{V}_0$

What is the final velocity? $\vec{V}_f$

<u>Energy Conservation</u> $E_1 = E_2$

$$K_1 + V_1 = K_2 + V_2$$

① refers to the situation on the table

② refers to the situation on the ground.

$$\frac{1}{2} \cancel{m} v_0^2 + \cancel{m} g h = \frac{1}{2} \cancel{m} v_f^2$$

$$\Rightarrow \quad v_f^2 = v_0^2 + 2gh$$

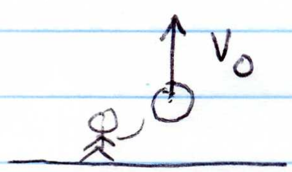
$$\boxed{v_f = \sqrt{v_0^2 + 2gh}}$$

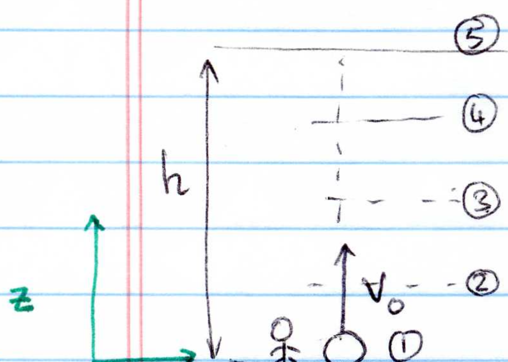
m does not matter in determining V

(4) Throwing a ball up in the air with velocity $\vec{V}_0$

How high does it go?

What is the velocity when it returns to the ground?





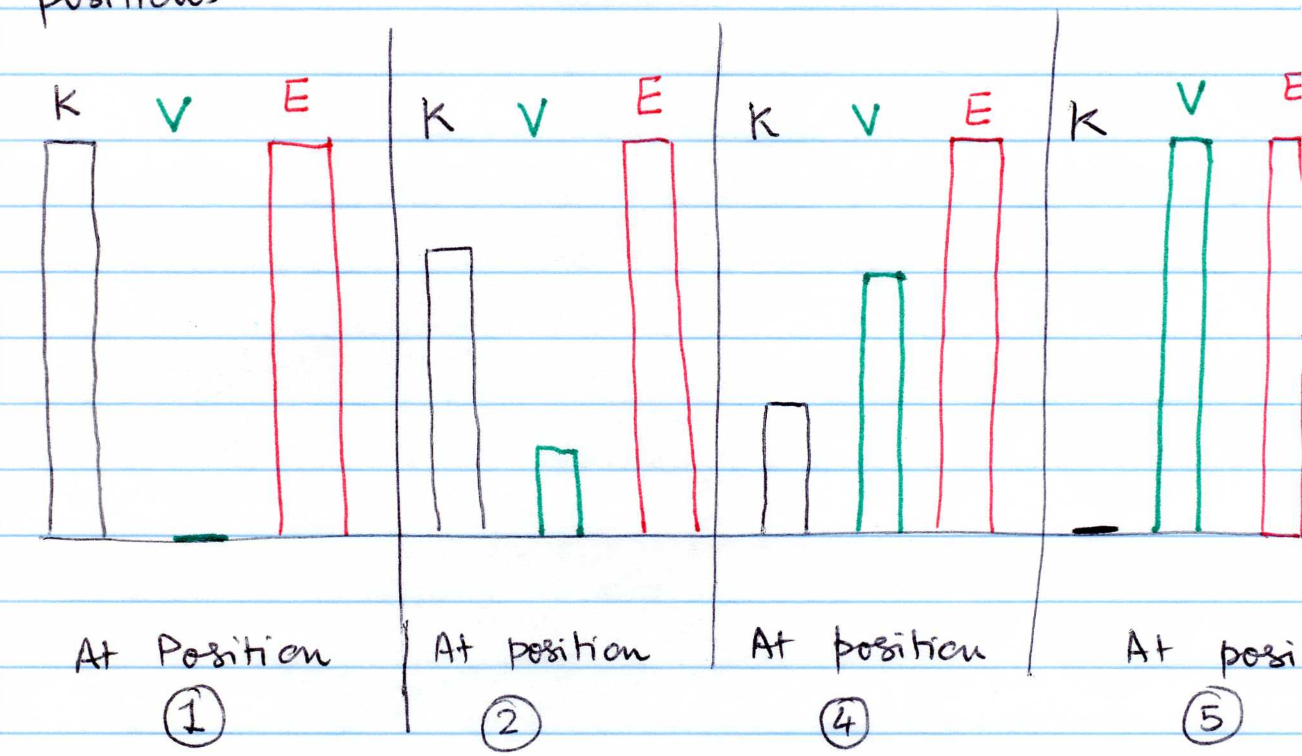
On the ground the ball has kinetic energy $= \frac{1}{2} m V_0^2$

and potential energy $= 0$

As the balls goes up it gains potential enegy mgz and consequently it must lose kinetic energy to keep the total energy constant

$$E = K + V$$

let us draw these quantities at different positions



As the ball moves up the energy sloshes between kinetic & potential energies keeping the total energy constant.

At the highest point all the energy is potential energy and the ball has come to a halt.

$$\underbrace{0 + mgh}_{\text{at position 5}} = \underbrace{\tfrac{1}{2} m v_0^2 + 0}_{\text{at position 1}} \Rightarrow \boxed{h = \frac{v_0^2}{2g}}$$

What is the velocity of the ball when it returns?

We can play the movie backwards.

Once again energy conservation dictates that

$$V_f = V_0.$$

More explicitly, the potential energy of the ball at 5 will get converted to kinetic energy when the ball returns

$$\frac{1}{2} m V_f^2 = mgh$$

$$\Rightarrow \quad V_f^2 = 2gh$$

We showed that $h = \dfrac{V_0^2}{2g}$

$$\Rightarrow \quad V_f^2 = \cancel{2g}\,\frac{V_0^2}{\cancel{2g}} = V_0^2 \quad \Rightarrow \quad |\vec{V}_f| = |\vec{V}_0|$$

Part II

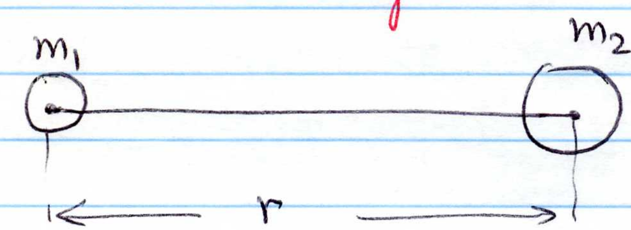
Potential Energy due to gravitational interaction

negative sign
$\Rightarrow$ always attractiv

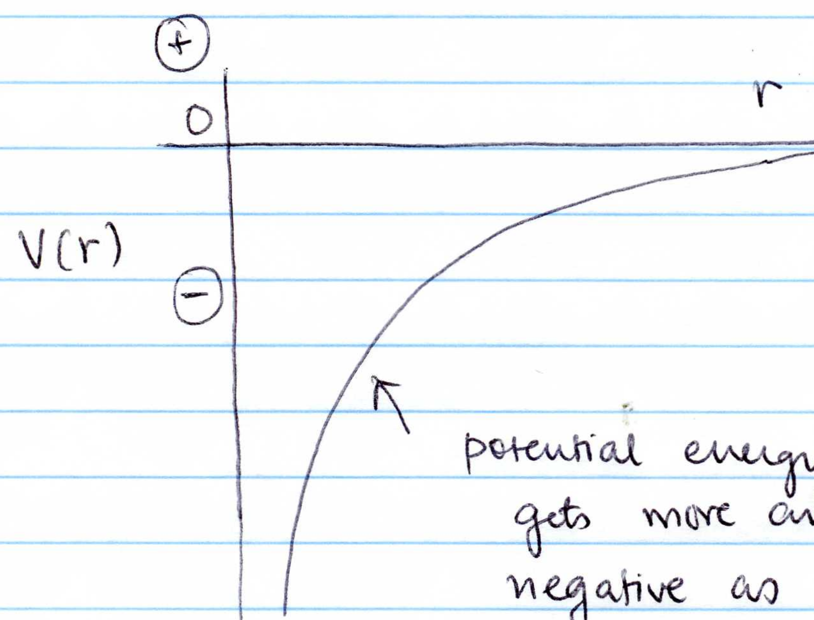
$$\boxed{V(r) = -\frac{G m_1 m_2}{r}}$$

Reference at
$r \Rightarrow \infty$ where $V(r) \rightarrow$

$G = 6.67 \times 10^{-11} \; \frac{J\,m}{kg^2}$



distance between two masses



goes to zero as the distan between masse gets very lar

potential energy gets more and more negative as masses come closer.

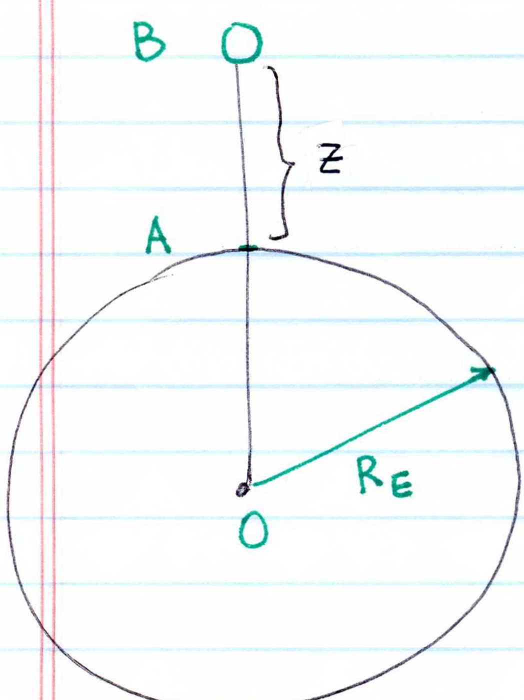
How is $V(r) = -\frac{G m_1 m_2}{r}$ related to $V(z) = m$

Let $m_1 = M$ mass of earth

$m_2 = m$ mass of object

R_E = radius of earth

V_A = potential energy of object when it is on the earth's surface



$$V_A = -\frac{GMm}{R_E} \quad \text{— (i)}$$

V_B = potential energy of object when it is a distance z above the surface

$$= -\frac{GMm}{R_E + z} \quad \text{(ii)}$$

$$\underbrace{V_B - V_A}_{\text{difference in potential energy}} = -\frac{GMm}{R_E + z} + \frac{GMm}{R_E} \quad \text{(iii)}$$

Now the size (radius) of the earth is ~ 6000 km.
And say we throw a ball up as hard a[s]
we can and it goes up ~ 100 m
or even 1000 m ≃ 1 Km

$$\frac{z}{R_E} \sim \frac{1\,\text{Km}}{6000\,\text{Km}} \sim 10^{-4} \ll 1$$

We can therefore simplify (iii) by utilizing this fact that $z/R_E \ll 1$

We will do some algebra to simplify

$$V_B - V_A = \frac{GMm}{R_E}\left[1 - \frac{R_E}{R_E + z}\right]$$

here $R_E + z = R_E\left(1 + \frac{z}{R_E}\right)$

$$= \frac{GMm}{R_E}\left[1 - \frac{\cancel{R_E}}{\cancel{R_E}(1 + z/R_E)}\right]$$

Taylor's Expansion (see notes later)

$$\frac{1}{1+\alpha} \simeq 1 - \alpha + \text{(higher order terms in}$$

$\alpha \ll 1$ → approximate if I keep on[ly] terms upto α.

e.g $\frac{1}{1.001} \simeq 0.99..$

$$V_B - V_A \simeq \frac{GMm}{R_E}\left[1 - \left(1 - z/R_E\right)\right]$$

$$= \frac{GMm}{R_E}\,\frac{z}{R_E}$$

$$\Delta V = m\left(\frac{GM}{R_E^2}\right) z = mgz$$

Define $g = \frac{GM}{R_E^2}$

plug in the universal constant G, Mass of ear
radius of earth and show that $g = 9.8\ m/s$