N5 Statics

The main question we want to address here is "Will this object, given the forces acting on it, move?"

To answer we have to determine the net force acting on the object and the net torque.

$$\vec{F}_{net} = \underbrace{\sum_i \vec{F}_i}_{\text{vector sum of all the forces}}$$

$$\vec{F}_{net} = m\vec{a}_{CM}$$

If $\vec{F}_{net} = 0 \Rightarrow \vec{a}_{CM} = 0 \Rightarrow \vec{V} =$ constant vector
(no change in magnitude or direction)

An object that moves as constant velocity can be viewed from a reference frame moving with the same velocity $\vec{v}$. From this so called "inertial" reference frame the object will be stationary.

$$\vec{\tau}_{net} = \underbrace{\sum_i \vec{\tau}_i}_{\text{vector sum}} = \sum_i \vec{r}_i \times \vec{F}_i$$

vector sum
of all the torques acting about a chosen origin

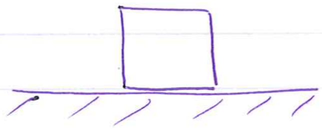
$$\vec{\tau}_{net} = \frac{d\vec{L}}{dt}$$

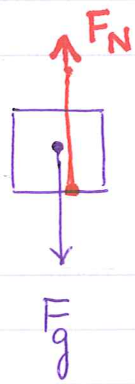
If $\vec{\tau}_{net} = 0 \Rightarrow \frac{d\vec{L}}{dt} = 0$

If a system is in STATIC EQUILIBRIUM there are no net forces and no net torques acting on it.

Example 1

Object on a table





$F_g = mg$ is the force of gravity acting downwards "towards center of the earth".

F_N = normal force upward

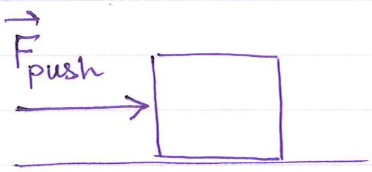
If the object is stationary (i.e not sinking into the table or flying off in the sky) then the two forces balance each other

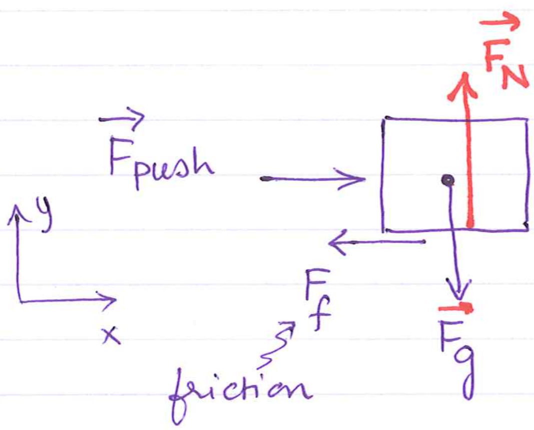
$$\vec{F}_g + \vec{F}_N = 0 \Rightarrow \vec{F}_N = -\vec{F}_g$$

$$\text{or} \quad F_N = F_g$$

magnitude of F_N equals magnitude of F_g but their directions are opposite.

Example 2 Pushing a block on a table





(Note : $\vec{F}_N$ passes through CM. I have offset it just to show it clearly)

$$\vec{F}_g + \vec{F}_N + \vec{F}_f + \vec{F}_{push} = \vec{F}_{net}$$

$$\begin{pmatrix} 0 \\ -F_g \end{pmatrix} + \begin{pmatrix} 0 \\ F_N \end{pmatrix} + \begin{pmatrix} -F_f \\ 0 \end{pmatrix} + \begin{pmatrix} F_{push} \\ 0 \end{pmatrix} = \begin{pmatrix} F_{net,x} \\ F_{net,y} \end{pmatrix}$$

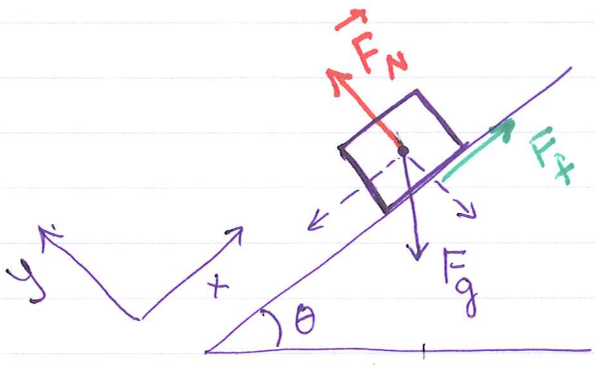
Equating the x and y components to zero separately gives

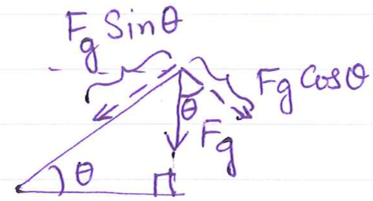
$$F_N - F_g = F_{net,y} = 0$$ (block is not moving along $\hat{y}$)

$$F_{push} - F_f = F_{net,x} = m a_x$$

If $F_{push} > F_f$ the block will accelerate along x.

Example 3 : Block on a Slope





Decompose F_g along the slope and perpendicular to the slope. It now helps to set up a co-ordinate system that is rotated as shown.

$$\begin{pmatrix} -F_g \sin\theta \\ -F_g \cos\theta \end{pmatrix} + \begin{pmatrix} F_f \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ F_N \end{pmatrix} = \begin{pmatrix} F_{net,x} \\ F_{net,y} \end{pmatrix}$$

$$-F_g \sin\theta + F_f = F_{net,x}$$

$$F_N = F_g \cos\theta$$ ($F_{net,y} = 0$ because block remains on the slope)

$\Rightarrow$ If $F_g \sin\theta > F_f$ the block will slide down.

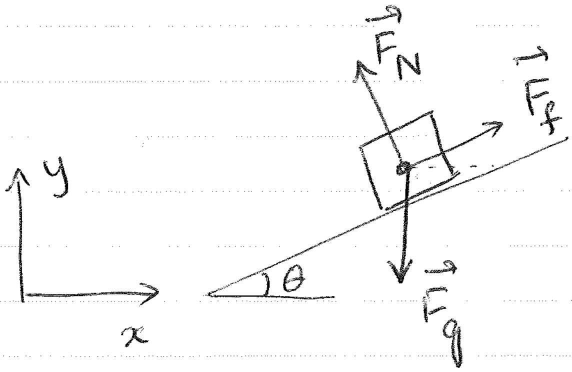
In equilibrium

$$\boxed{\begin{aligned} F_N &= F_g \cos\theta \\ F_f &= F_g \sin\theta \end{aligned}}$$

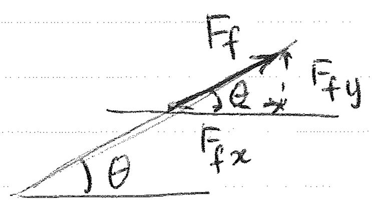
Question in class :

Can we use the usual $x-y$ axes rather than the rotated axes.

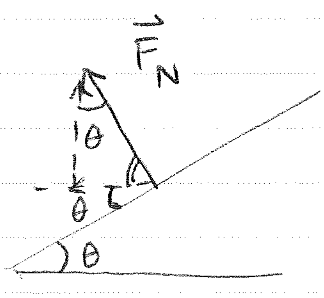
Sure we can:



Decompose $\vec{F}_f$ and $\vec{F}_N$ along $\hat{x}$ and $\hat{y}$



$$\vec{F}_f = \begin{pmatrix} F_f \cos\theta \\ F_f \sin\theta \end{pmatrix}$$



$$\vec{F}_N = \begin{pmatrix} -F_N \sin\theta \\ F_N \cos\theta \end{pmatrix}$$

$$\sum_i \vec{F}_i = 0 \Rightarrow \begin{pmatrix} F_f \cos\theta \\ F_f \sin\theta \end{pmatrix} + \begin{pmatrix} -F_N \sin\theta \\ F_N \cos\theta \end{pmatrix} + \begin{pmatrix} 0 \\ -F_g \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

Equating the x and y components separately to zero :

$$F_f \cos\theta - F_N \sin\theta = 0 \Rightarrow F_f = F_N \tan\theta \quad -(1)$$

$$F_f \sin\theta + F_N \cos\theta = F_g . \quad -(2)$$

We want to solve for both F_f and F_N in terms of F_g and θ.

Substitute for F_f from Eq. (1) in Eq. (2)

$$(F_N \tan\theta) \sin\theta + F_N \cos\theta = F_g$$

$$F_N \left[\frac{\sin^2\theta}{\cos\theta} + \cos\theta \right] = F_g$$

$$F_N \left[\frac{\overbrace{\sin^2\theta + \cos^2\theta}^{=1}}{\cos\theta} \right] = F_g \Rightarrow \boxed{F_N = F_g \cos\theta}$$

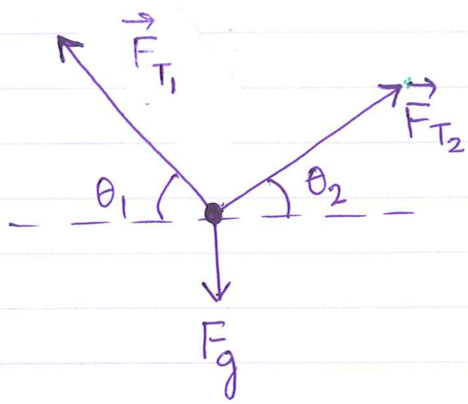
From Eq. (1)

$$F_f = F_N \tan\theta = F_g \cos\theta \cdot \frac{\sin\theta}{\cos\theta}$$

$$\Rightarrow \boxed{F_f = F_g \sin\theta}$$

Same as in the other approach where we defined our axes along and perpendicular to the plane (slope)

Example 4 Climber hanging by two ropes



$$\begin{pmatrix} -F_{T_1}\cos\theta_1 \\ +F_{T_1}\sin\theta_1 \end{pmatrix} + \begin{pmatrix} F_{T_2}\cos\theta_2 \\ F_{T_2}\sin\theta_2 \end{pmatrix} + \begin{pmatrix} 0 \\ -F_g \end{pmatrix} = \begin{pmatrix} F_{net,x} \\ F_{net,y} \end{pmatrix}$$

Assume the rock climber is stationary $\Rightarrow F_{net,x} = 0$ & $F_{net,y} = 0$

$$\Rightarrow -F_{T_1}\cos\theta_1 + F_{T_2}\cos\theta_2 = 0$$

$$\Rightarrow \boxed{\frac{F_{T_1}}{F_{T_2}} = \frac{\cos\theta_2}{\cos\theta_1}}$$ from x - eq.

$$F_{T_1}\sin\theta_1 + F_{T_2}\sin\theta_2 = F_g$$

Substitute for F_{T_1} from above

$$\left(F_{T_2}\frac{\cos\theta_2}{\cos\theta_1}\right)\sin\theta_1 + F_{T_2}\sin\theta_2 = F_g$$

$$\Rightarrow \quad F_{T_2} \cos\theta_2 \tan\theta_1 + F_{T_2} \sin\theta_2 = F_g$$

$$\Rightarrow \quad F_{T_2}\left[\cos\theta_2 \tan\theta_1 + \sin\theta_2\right] = F_g$$

$$\Rightarrow \quad \boxed{F_{T_2} = \frac{F_g}{\cos\theta_2 \tan\theta_1 + \sin\theta_2}}$$

$$\Rightarrow \quad F_{T_1} = F_{T_2} \frac{\cos\theta_2}{\cos\theta_1}$$

$$= \frac{F_g}{\left(\cos\theta_2 \tan\theta_1 + \sin\theta_2\right)} \frac{\cos\theta_2}{\cos\theta_1}$$

$$= \frac{F_g \cos\theta_2}{\cos\theta_2 \sin\theta_1 + \sin\theta_2 \cos\theta_1}$$

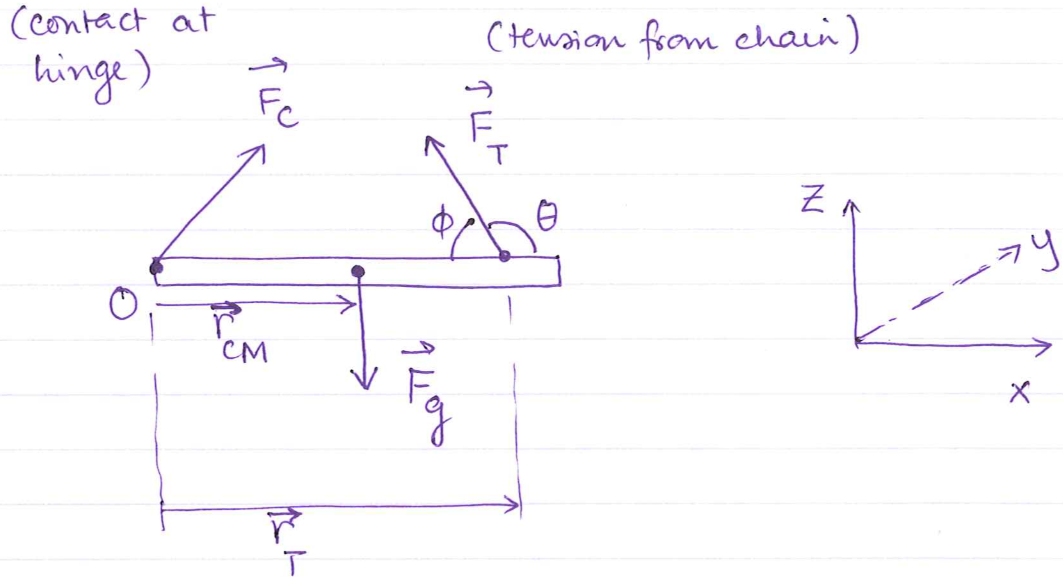
use

$\sin(A+B) =$

$\sin A \cos B + \cos A \sin B$

$$\boxed{F_{T_1} = \frac{F_g \cos\theta_2}{\sin(\theta_1 + \theta_2)}}$$

Example N5.5 drawbridge with a chain.



The following information is given:

M = mass of drawbridge

L = length of drawbridge

r_T = distance of chain from hinge O.

θ = angle of chain & drawbridge

Find F_T and $\vec{F}_c$

Step 1: Balance forces (drawbridge is at rest)

$$\begin{pmatrix} 0 \\ -F_g \end{pmatrix} + \begin{pmatrix} -F_T \cos\phi \\ F_T \sin\phi \end{pmatrix} + \begin{pmatrix} F_{c,x} \\ F_{c,y} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\Rightarrow \quad F_{c,x} = F_T \cos\phi$$

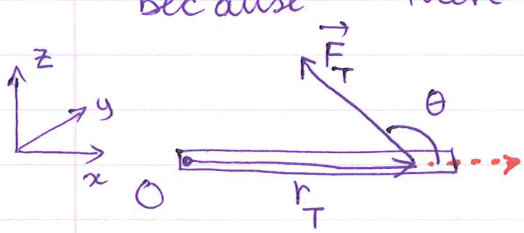
$$F_{c,y} = F_g - F_T \sin\phi$$

This shows that by balancing forces we can find $F_{c,x}$ and $F_{c,y}$ in terms of the (so far unknown) F_T.

We need one more equation:

Balance Torques: about the hinge O.

$\vec{F}_c$ does not exert a torque about O because there is no lever arm.



Torque due to chain

$$\vec{r}_T \times \vec{F}_T = -\, r_T F_T \sin\theta \, \hat{y}$$

extend $\vec{r}_T$ along dotted line

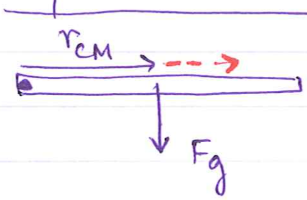
move $\vec{r}_T$ toward $\vec{F}_T$

screw moves out of the paper.

+y is defined as into the paper

(see axes figure)

Torque due to $\vec{F}_g$



$$\vec{r}_{CM} \times \vec{F}_g = + \, r_{CM} F_g \underbrace{\cancel{\sin 90^\circ}}_{=1} \hat{y}$$

$$= \frac{1}{2} L M g \, \hat{y}$$

$r_{CM} = \frac{L}{2}$

Balance torques along $\hat{y}$

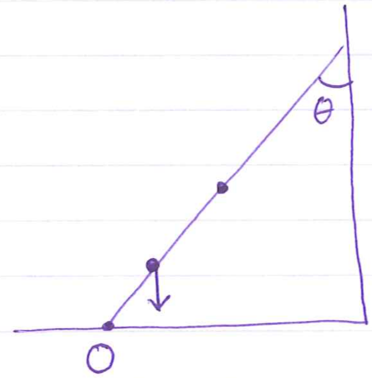
$$\Rightarrow \quad \vec{\tau}_{chain} + \vec{\tau}_{gravity} = 0$$

$$\Rightarrow \quad -r_T F_T \sin\theta + \frac{1}{2} L M g = 0$$

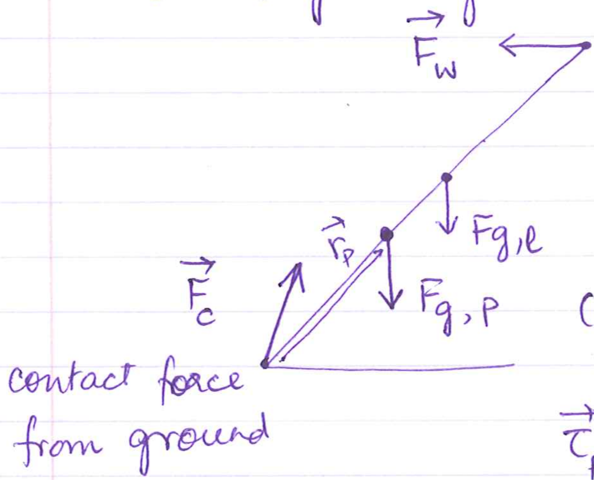
$$\Rightarrow \quad \boxed{F_T = \frac{L M g}{2 r_T \sin\theta}}$$

Now we have F_T, we can go back and solve for F_{cx} and F_{cy}.

N5B.4



Free Body diagram of ladder

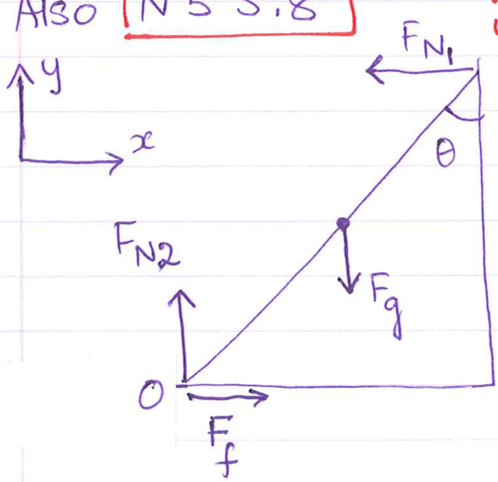


$F_{g,l}$ (gravity, due to ladder)

$F_{g,p}$ (gravity due to person)

$$\vec{\tau}_p = \vec{r}_p \times \vec{F}_g \quad \text{(clockwise)}$$

Also $\boxed{\text{N5S.8}}$ ← we are given frictionless surface with wall



$\vec{F}_{N_1}$ = normal force from wall

$\vec{F}_{N2}$ = normal force from ground

$$\vec{F}_{N_1} + \vec{F}_{N_2} + \vec{F}_g + \vec{F}_f = 0$$

$$\begin{pmatrix} -F_{N_1} \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ F_{N_2} \end{pmatrix} + \begin{pmatrix} 0 \\ -F_g \end{pmatrix} + \begin{pmatrix} F_f \\ 0 \end{pmatrix} = 0$$

$F_{N_1} = F_f$

$F_{N_2} = F_g$

Now use torque equations: Pick the origin about O. (ground)

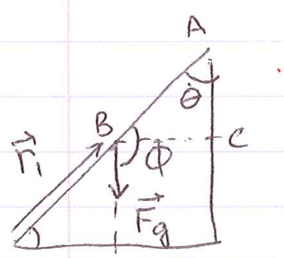
$$\underbrace{\vec{r}_1 \times \vec{F}_g}_{\text{clockwise}} + \underbrace{\vec{r}_2 \times \vec{F}_{N_1}}_{\text{anticlockwise}} = 0.$$

($\vec{F}_f$ and $\vec{F}_{N_1}$ don't exert any torque because the lever arm is zero)

$|\vec{r}_1| = L/2$

$|\vec{r}_2| = L$

$$|\vec{r}_1 \times \vec{F}_g| = \frac{L}{2} F_g \sin\phi = \frac{L}{2} F_g \sin(180 - \theta) = \frac{L}{2} F_g \sin\theta.$$



How is ϕ related to θ? ↑ given

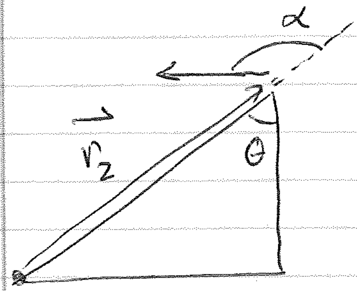
$\angle ABC = 90^\circ - \theta$

$\phi = 90 + \angle ABC$

$= 90 + 90 - \theta$

$= 180 - \theta$

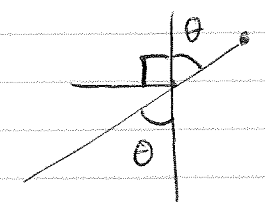
$$|\vec{r}_2 \times \vec{F}_{N_1}| = r_2 F_{N_1} \sin\alpha$$



$$= r_2 F_{N_1} \sin(90+\theta)$$

$$= r_2 F_{N_1} \cos\theta \quad = L F_{N_1} \cos\theta$$

How is α related to θ ?



$$\alpha = 90 + \theta$$

New putting the two torques together

$$\frac{L}{2} F_g \sin\theta = L F_{N_1} \cos\theta$$

$$\Rightarrow \boxed{F_{N_1} = \frac{1}{2} F_g \tan\theta} = F_f$$

$$\boxed{F_{N_2} = F_g}$$