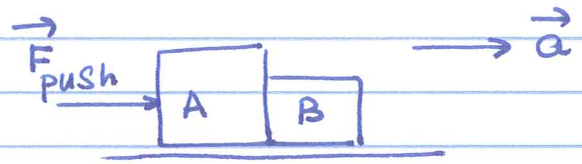
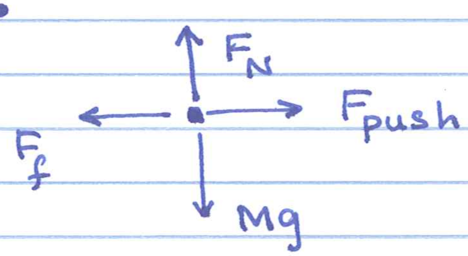
Coupled Objects N7



Two blocks are pushed by some force $\vec{F}_{push}$

What is the acceleration $\vec{a}$?

I Since the two blocks are acting as a rigid body we can solve by considering the COM of the whole unit $M = m_A + m_B$



$$F_{push} - F_f = Ma \qquad F_N = Mg.$$

$$F_f = \mu_k F_N = \mu_k M g$$

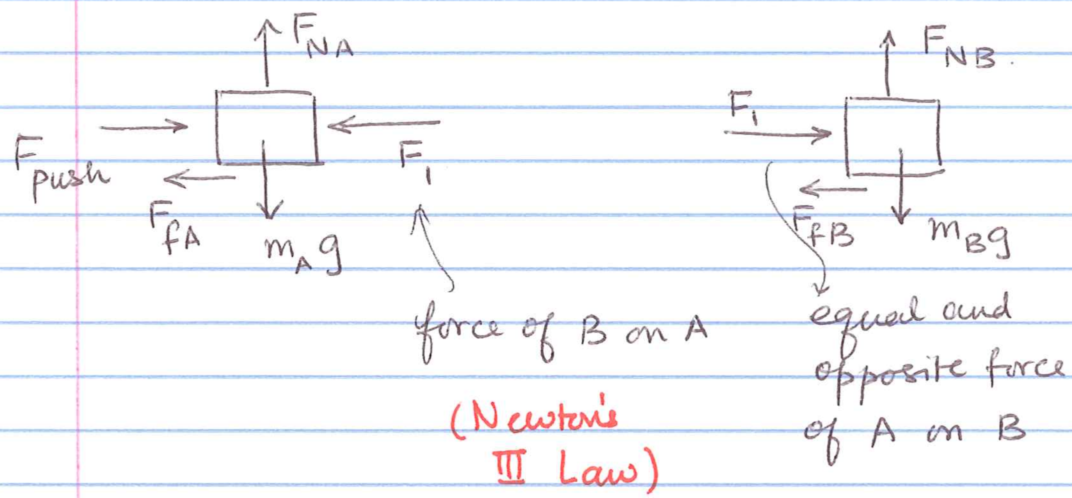
μ_k = coefft. of kinetic friction

$$\Rightarrow \quad Ma = F_{push} - \mu_k M g$$

$$\Rightarrow \quad \boxed{a = \frac{F_{push}}{M} - \mu_k g}$$

II What if the two blocks are made of different materials with coefficients of friction μ_A and μ_B?

Draw free body diagrams for each block:



IMPORTANT

The two blocks must accelerate with the same $\vec{a}$ so as not to get disengaged.

$$F_{NA} = m_A g$$

$$F_{push} - F_1 - F_{fA} = m_A a$$

$$F_{fA} = \mu_A F_{NA} = \mu_A m_A g$$

$$\Rightarrow F_{push} - F_1 - \mu_A m_A g = m_A a$$

$$\Rightarrow F_1 = F_{push} - m_A(\mu_A g + a)$$

$$F_{NB} = m_B g$$

$$F_1 - F_{fB} = m_B a$$

$$F_{fB} = \mu_B F_{NB}$$

$$= \mu_B m_B g$$

$$F_1 = m_B a + F_{fB}$$

$$= m_B(a + \mu_B g)$$

Equating the two expressions for F_1

$$F_{push} - m_A(\mu_A g + a) = m_B(a + \mu_B g)$$

$$\Rightarrow \quad m_A a + m_B a = F_{push} - (m_A \mu_A + m_B \mu_B) g$$

$$\Rightarrow \quad \boxed{a = \frac{F_{push}}{M} - \frac{m_A \mu_A + m_B \mu_B}{M} g.}$$

$M = m_A + m_B$

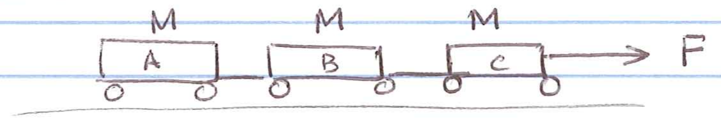
We recover the previous expression in I when

$$\mu_A = \mu_B = \mu_k$$

~~and~~

K&K

III Freight Train



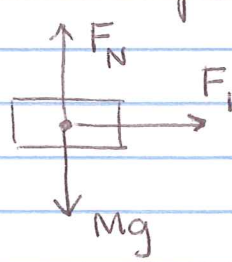
Three freight cars of mass M are pulled by a force F by the engine. Friction is negligible. Find forces on each car.

First consider the whole system:

$$F = M_{total}\, a = 3M\, a$$

$$\Rightarrow a = \frac{F}{3M}$$

Free Body diagram of last car (A).

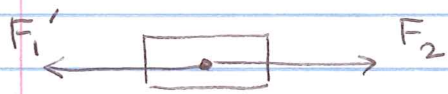


F_1 — force exerted by (B) on (A)

vertical acceleration is zero so $F_N = Mg$

$$F_1 = Ma = M\left(\frac{F}{3M}\right) = \frac{F}{3}$$

Consider the central car (B)



The vertical forces once again balance

F_1' = force exerted by A on B

$= F_1$ by Newton's III Law.

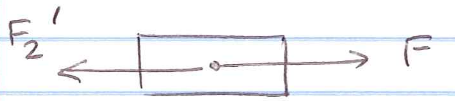
$$= \frac{F}{3}$$

$$F_2 - F_1' = Ma$$

$$F_2 = Ma + F_1' = M\left(\frac{F}{3M}\right) + \frac{F}{3}.$$

$$F_2 = \frac{2F}{3}$$

Consider the first car (c)



$$F - F_2' = Ma$$

By Newton's III Law $F_2' = F_2 = \frac{2F}{3}$

$$F - F_2' = \frac{F}{3}$$

$\Rightarrow$ Each car experiences a net force $\frac{F}{3}$ to the right which is responsible for the acceleration of $a = \frac{F}{3M}$.

<u>General solution</u>: Consider a string of N cars. each of mass M pulled by force F

$$a = \left(\frac{F}{NM}\right)$$

To find the force F_n pulling the last n cars we must have the force F_n giving the mass nM an acceleration $\frac{F}{NM}$.

$$F_n = (nM)\left(\frac{F}{NM}\right)$$

$$F_n = \frac{n}{N} F$$

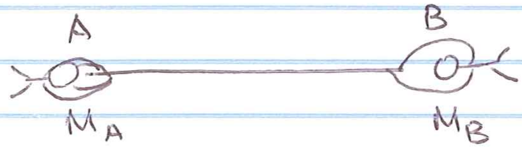
$\Rightarrow$ The force on each car ~~equals~~ is proportional to the # of cars pulled.

IV Strings

Tug of War

astronaut's tug of war

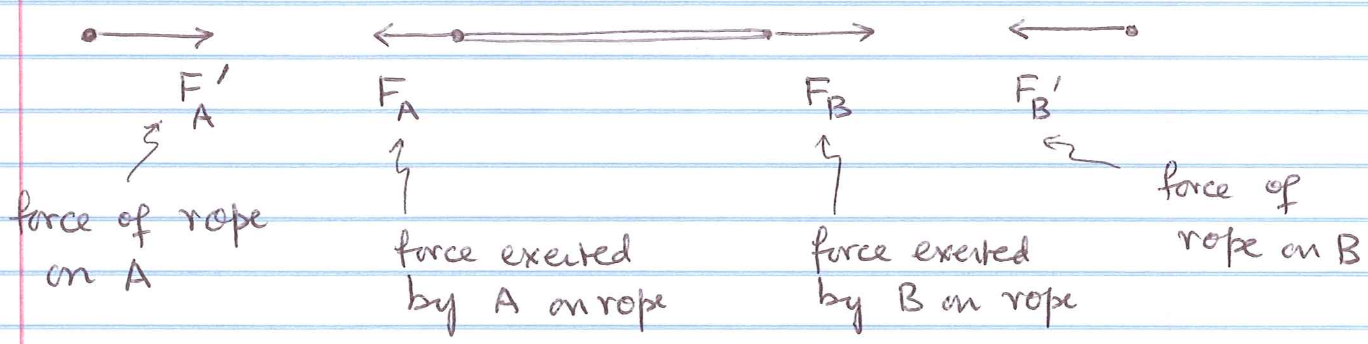
M_r = mass of rope.



Two astronauts Alice & Bob initially at rest in free space pull on either end of a rope. A pulls with a force F_A and B pulls with a maximum force F_B.

Find their motion if both Alice and Bob pull as hard as they can.

Free Body diagrams of A, B and rope



By Newton's III Law:

$$F_A' = F_A \qquad F_{B'} = F_B$$

Equation of motion for rope :

$$\sum \vec{F}_i = M_r \, a_r$$

$$F_B - F_A = M_r \, a_r \qquad \text{— (1)}$$

Equation of motion of Alice :

$$F_A = M_A \, a_A \qquad (2)$$

Equation of motion of Bob

$$-F_B = M_B \, (-a_B)$$

$$F_B = M_B \, a_B \qquad \text{— (3)}$$

From Eq. (1), (2) & (3)

$$\boxed{M_B \, a_B - M_A \, a_A = M_R \, a_R}$$

If the mass of the rope is negligible

$$\Rightarrow \quad F_B = F_A = F$$

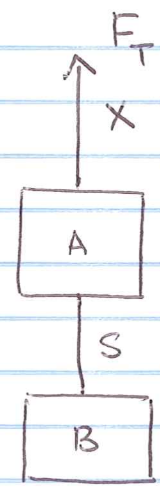
The astronauts each pull with the <u>same</u> force.

The accelerations of the two astronauts is

$$a_A = \frac{F}{M_A} \quad ; \quad a_B = \frac{F}{M_B}$$

they are accelerating in opposite directions

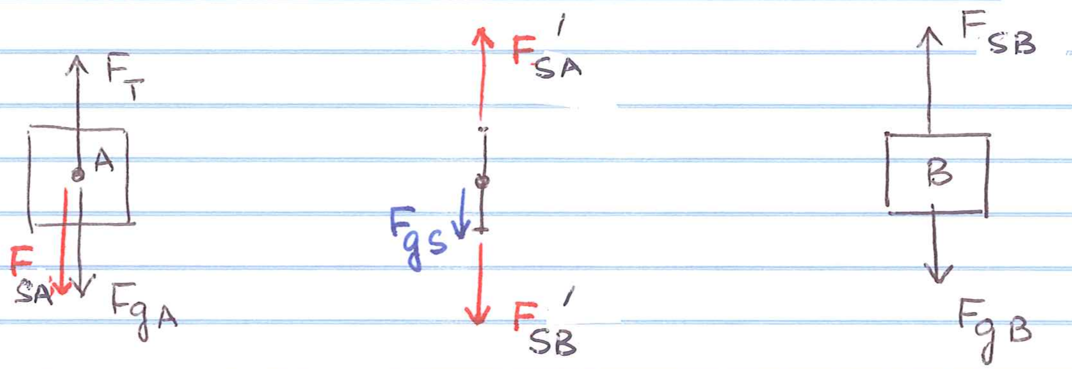
N7.3



Two blocks connected by an internal string S vertically pulled by an external string X

Find the forces exerted by the string on the blocks and the acceleration of the system

Free body diagram of blocks A, B and internal string S.



By Newton's III Law: $F_{SA} = F'_{SA}$

$F_{SB} = F'_{SB}$

Assume the objects move vertically with the same acceleration.

$\uparrow z$

$$\sum_i \vec{F}_i^{\,A} = M_A\, a_z$$

$$F_T - F_{gA} - F_{SA} = M_A\, a_z \qquad (1)$$

$$\sum_i \vec{F}_i^{\,B} = M_B\, a_z$$

$$F_{SB} - F_{gB} = M_B\, a_z \qquad (2)$$

$$\sum_i \vec{F}_i^{\,S} = M_S\, a_z$$

$$F_{SA} - F_{SB} - F_{gS} = M_S\, a_z \qquad (3)$$

There are two unknown forces F_{SA} and F_{SB} and one unknown acceleration a_z

From (1) $$F_{SA} = F_T - F_{gA} - M_A\, a_z \qquad (4)$$

From (2) $$F_{SB} = F_{gB} + M_B\, a_z \qquad (5)$$

Plug (4) & (5) in (3)

$$F_T - F_{gA} - M_A a_z - F_{gB} - M_B a_z - F_{gS} = M_S a_z$$

$$\Rightarrow F_T - F_{gA} - F_{gB} - F_{gS} = (M_A + M_B + M_S)\, a_z$$

$$\Rightarrow \boxed{a_z = \frac{F_T - (M_A + M_B + M_S)g}{M_A + M_B + M_S}} \qquad (6)$$

$$M_{total} = M_A + M_B + M_S$$

$$\boxed{a_z = \frac{F_T - M_{total}\, g}{M_{total}}} \quad \text{— (7)}$$

We could have written this down right away without breaking up the full system into its parts.

But now if we want to calculate F_{SA} and F_{SB} then we do want to analyse the problem in parts.

$$\text{Eq.(4)} \Rightarrow F_{SA} = F_T - F_{gA} - M_A\left[\frac{F_T - M_{total}\, g}{M_{total}}\right]$$

$$= F_T\left(1 - \frac{M_A}{M_{Total}}\right) - \cancel{M_A g} + \cancel{M_A g}.$$

$$\boxed{F_{SA} = F_T\left(\frac{M_B + M_S}{M_{Total}}\right)} \quad \text{— (8)}$$

$$F_{SB} = M_B\, g + M_B\left(\frac{F_T - M_{total}\, g}{M_{total}}\right)$$

$$F_{SB} = \frac{M_B}{M_{total}} F_T + \cancel{M_B g} - \cancel{M_B g}$$

$$\boxed{F_{SB} = \frac{M_B}{M_{total}} F_T} \quad \text{— (9)}$$

Assume string is massless:

$$a_z = \frac{F_T - (M_A + M_B)\,g}{M_A + M_B}$$

Eq. (3) $\Rightarrow$ $F_{SA} - F_{SB} = 0$

check:

Eq. (8) $\Rightarrow$ $F_{SA} = F_T \left(\frac{M_B}{M_A + M_B} \right)$

$$F_{SB} = F_T \frac{M_B}{M_A + M_B}$$

$\Rightarrow$ $F_{SA} - F_{SB} = 0$ ✓