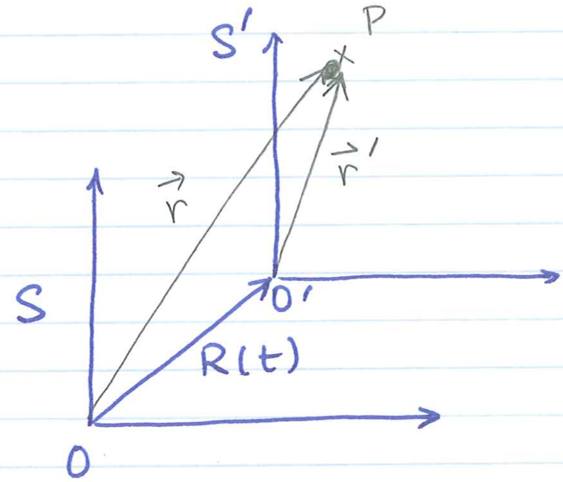
# Noninertial Frames




S = inertial frame (moving at constant velocity)

S' = noninertial frame (accelerating frame)

we will assume uniformly accelerating

$$\vec{u}(t) = \frac{d\vec{R}}{dt}$$

$$\vec{A} = \frac{d\vec{u}}{dt} = \text{const.}$$

$$\vec{r} = \vec{r}\,' + \vec{R}(t)$$

position of a point P measured in S

position of same point P measured in S'

vector connecting the origins O'O.

In general the point P could be moving in time

$$\vec{r}(t) = \vec{r}\,'(t) + \vec{R}(t)$$

$$\vec{v}(t) = \vec{v}\,'(t) + \vec{u}(t)$$ — velocity of S' relative to S

$$\vec{a}(t) = \vec{a}\,'(t) + \vec{A}$$ — acceleration of S' relative to S.

measured in S

measured in S'

$$\boxed{\text{Newton's Law} \quad \vec{F} = m\vec{a} \quad \text{in inertial frame } S.}$$

$$= m\vec{a}\,' + m\vec{A}$$

$$= \vec{F}\,' + m\vec{A}$$

$$\boxed{\vec{F} = \underbrace{\vec{F}\,' - \vec{F}_{fict}}}$$

Define fictitious force

$$\vec{F}_{fict} = -m\vec{A}$$

In the inertial frame we will talk about real forces

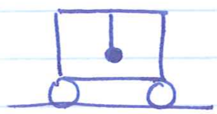
In the non inertial frame we will talk about real forces and fictitious forces due to acceleration.

The way we will use this equation is as follows: In the $S'$ frame we will write down all the forces $\vec{F}$ on an object <u>AND</u> add the fictitious force to it

$$\vec{F} + \vec{F}_{fict}$$

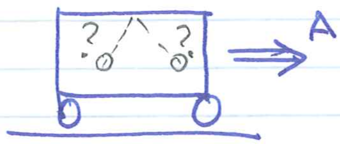
and set this equal to $m\vec{a}\,'$

## Example 1



Pendulum hangs in a car

Car is stationary.




Car accelerates at rate $\vec{A}$

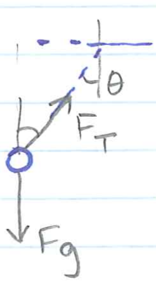
What is the angle $\theta$?

What is $F_T$?

Does pendulum move toward the front of the car?

or the back of the car?

| Inertial System S (road) | System accelerating with car non inertial S' |
|---|---|
| | |




$$\vec{a} = \vec{A}$$

$$F_T \cos\theta = F_g \Rightarrow F_T = \frac{F_g}{\cos\theta}$$

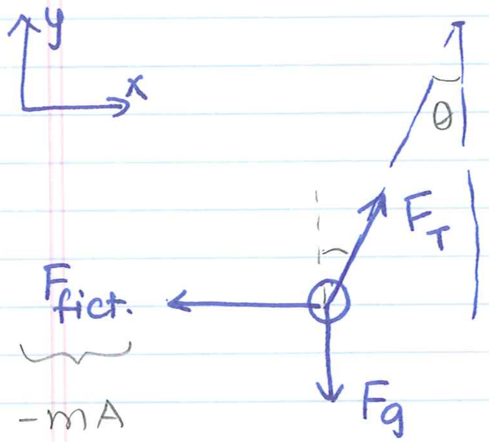
$$F_T \sin\theta = m A$$

$$\Rightarrow \tan\theta = A/g$$

$$F_T = m(g^2 + A^2)^{1/2}$$

$\underline{S'}$ Forces on pendulum:

(a) $F_g$ (b) $F_T$ (c) $F_{fict.}$




$\underbrace{F_{fict.}}$ $= -mA$

acts opposite to direction of $\vec{A}$

In this frame the pendulum is not accelerating

$$\vec{a}' = 0$$

The frame $S'$ is accelerating with respect to $S$ but since the pendulum is stationary in $S'$; $\vec{a}' = 0$ as measured in $S'$.

In $S'$ these three forces balance.

$$\vec{F} = \begin{pmatrix} 0 \\ -F_g \end{pmatrix} + \begin{pmatrix} F_T \sin\theta \\ F_T \cos\theta \end{pmatrix}$$

$$\vec{F}_{fict} = \begin{pmatrix} -mA \\ 0 \end{pmatrix}$$

$$\vec{F}' = \vec{F} + \vec{F}_{fict} = \begin{pmatrix} F_T \sin\theta - mA \\ -F_g + F_T \cos\theta \end{pmatrix} = 0 \quad (\because \vec{a}' = 0)$$

$$\Rightarrow F_g = F_T \cos\theta \qquad \left.\begin{aligned} F_T \sin\theta &= +mA \\ F_T \cos\theta &= F_g \end{aligned}\right] \Rightarrow \boxed{\tan\theta = \frac{A}{g}}$$

## Principle of Equivalence

$$\vec{F}_{fict} = -m\vec{A}$$

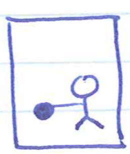
$\vec{F}_{fict}$ is indistinguishable from the force due to a uniform gravitational field $\vec{g} = -\vec{A}$

Both are constant forces proportional to $m$

$\vec{F}_{fict}$ acts at the COM.

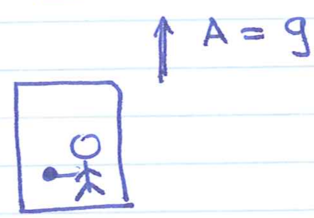
Consider two situations:

$\boxed{A}$



- Person in an elevator holding an apple
- Elevator is <u>not</u> moving
- Gravitational field $\vec{g}$

$\boxed{B}$




- person in an elevator holding an apple
- elevator is accelerating up at rate $g$
- No gravitational field

Person lets go of the apple.

What happens?

$y$

$\boxed{A}$

$\downarrow F_g$

$-F_g = ma$

$\Rightarrow a = -g$

apple accelerates along $-\hat{y}$ direction at rate $= g$

$\Rightarrow \vec{a} = -g\hat{y}$

$\boxed{B}$

$\downarrow F_{fict} = mA$

There is no gravitational force on B.

There is a fictitious force acting in the opposite direction to the acceleration.

$$\vec{F}' = \vec{F} + \vec{F}_{fict} = mA(-\hat{y})$$

$$= m\vec{a}'$$

$$\Rightarrow \vec{a}' = -A\hat{y}$$

If $A = g$

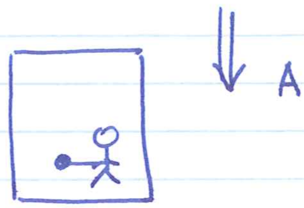
$$\vec{a}' = -g\hat{y}$$

It appears to the person in the upward accelerating elevator that the apple accelerates down at a rate $g$

## EQUIVALENCE PRINCIPLE

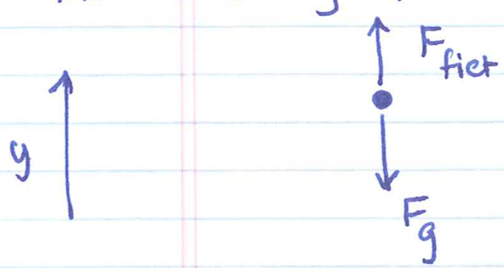
The laws of physics in a uniformly accelerating reference frame are identical to those in an inertial frame provided we introduce a fictitious force.

Another example:




- Person in an elevator holding an apple
- Elevator is accelerating down at rate A
- elevator is in a gravitational field g

As viewed by person




$$F_{ict} - F_g = mA - mg$$
$$= m(A-g)$$
$$= ma'$$

$$\Rightarrow \quad a' = A - g$$

if $A = g \Rightarrow a' = 0$

$\Rightarrow$ free fall in a gravitational field

If the person releases the apple it will float as if the elevator is motionless in free space.