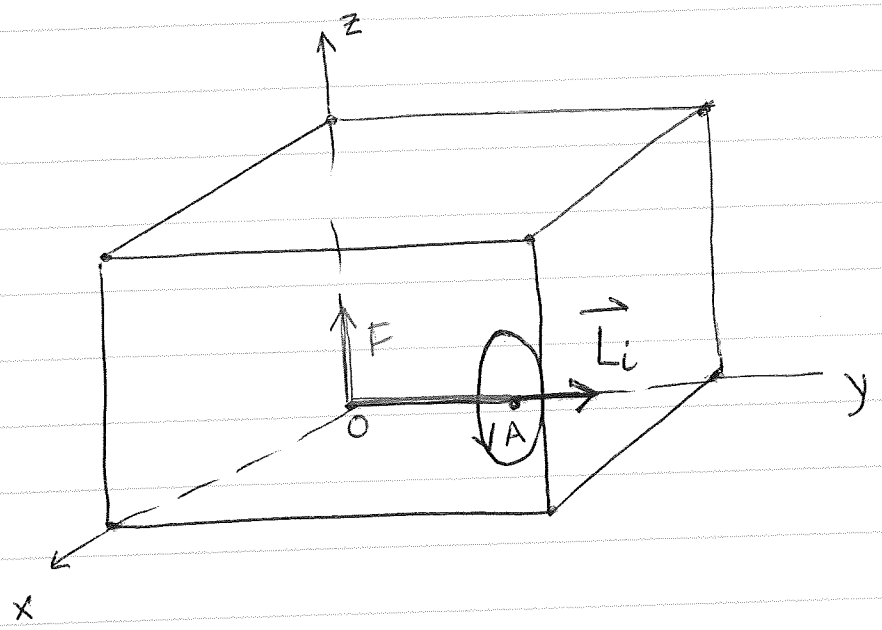
Torques & Spinning Wheels



Consider a wheel spinning with its angular momentum pointing along $\hat{y}$

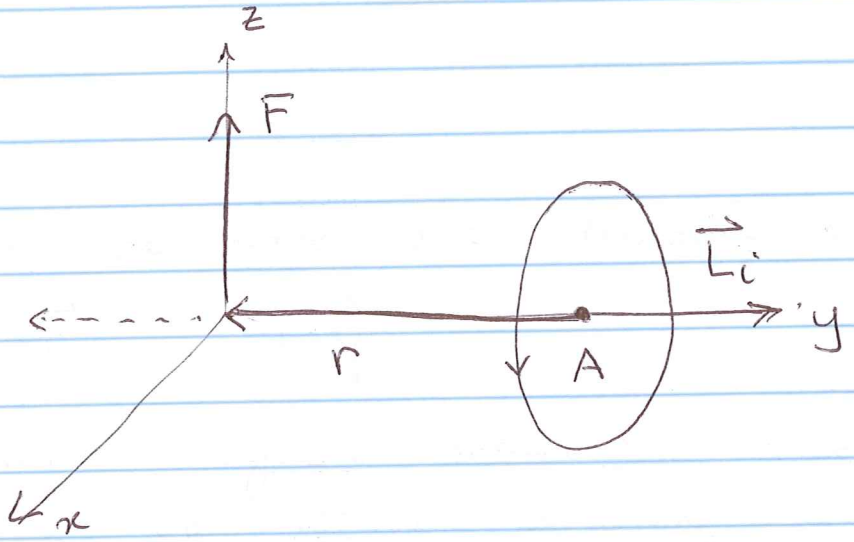
$$\vec{L}_{in} = L\,\hat{y}$$

Assume you are standing at O and holding the wheel by a rod or an axle.

Now assume you apply an upward force $\vec{F}$ as shown. This could be done for example by attaching a string to the rod at O and giving it an upward tug.

(a) Find out how the rotating wheel will be affected?

The force $\vec{F}$ results in a torque $\vec{\tau}$ on the wheel acting across a lever arm OA. Let me draw a bigger picture.



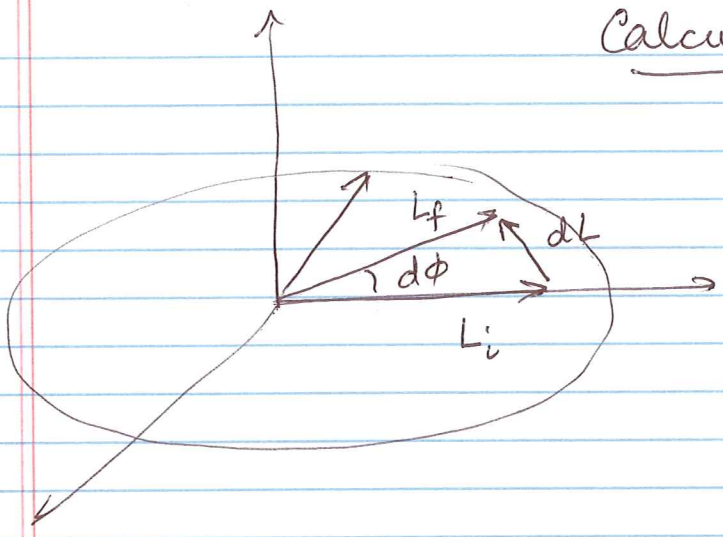
The torque is given by

$$\vec{\tau} = \vec{r} \times \vec{F}$$

To calculate this extend $\vec{r}$ along the dotted line and move the vector $\vec{r}$ towards $\vec{F}$. The screw moves into the board i.e. along the $-\hat{x}$ direction

$$\vec{\tau} = rF \sin\theta \, (-\hat{x})$$

Calculation of precession rate



Consider the movement of L in a small time dt.

$$\vec{\tau} = \frac{d\vec{L}}{dt} \Rightarrow |\vec{\tau}| = \frac{|d\vec{L}|}{dt}$$

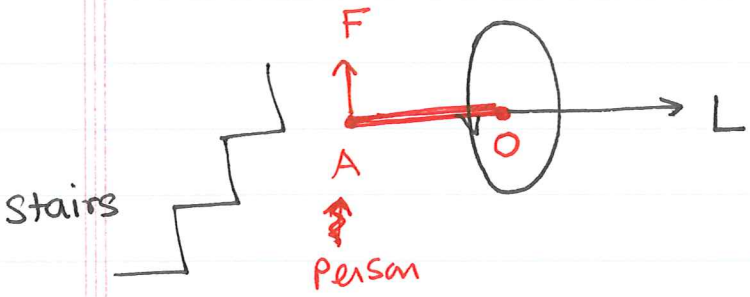
$$rF = L\frac{d\phi}{dt}$$

$\Rightarrow$ precession rate $\boxed{\frac{d\phi}{dt} = \frac{rF}{L} = \frac{rF}{I\omega}}$

Suit case & Wheel

assume wheel is held in right hand

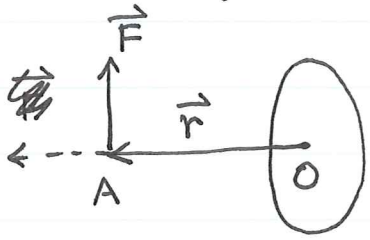
Person holding a spinning wheel goes up the stairs.



Which way does the wheel turn?

(1) As the person goes up the stairs, she gives the rod connected to the spinning wheel a tug in the vertical direction.

(2) This tug generates a torque about the axis of the spinning wheel.

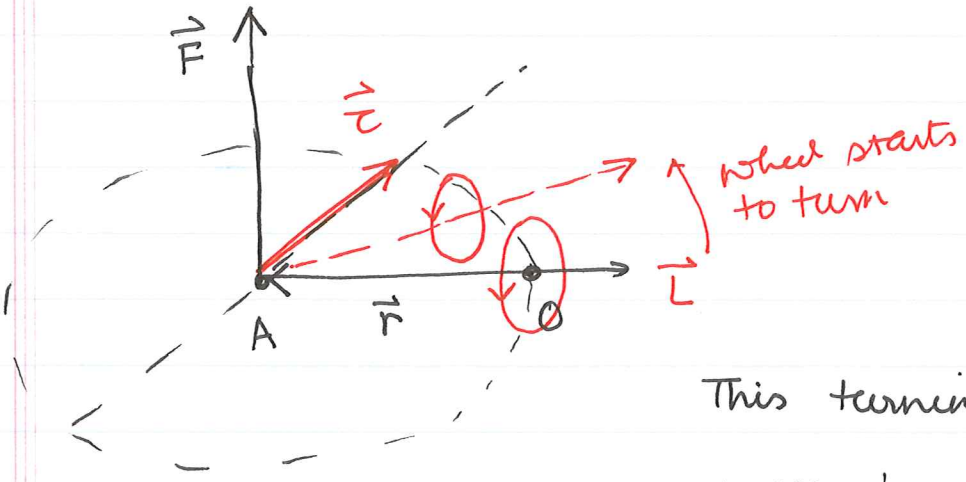


To find the torque extend $\vec{r}$ along dotted line, move $\vec{r}$ toward $\vec{F}$ and you see the screw moves into the paper

$\vec{\tau} = r F$ (into the paper)

(3) $\vec{\tau} = \frac{d\vec{L}}{dt}$

$\Rightarrow$ $\vec{L}$ will change in the horizontal plane and the wheel will move toward my front



This turning is the precession of the wheel.

If ~~you~~ the person continues to apply the force the wheel will precess around in that horizontal plane.