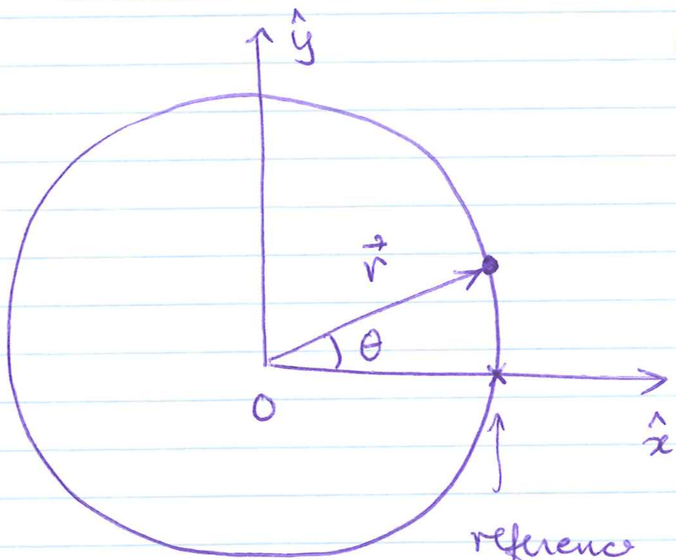
Oct. 2, 2012

Uniform Circular Motion



Uniform angular speed ω

ω = const (does not change in time)

r = radius of circle.

$$\boxed{\theta = \omega t} \quad \text{—— (1)}$$

reference position at $t=0$

$$\boxed{\vec{r} = \underbrace{r\cos(\omega t)}_{r_x}\hat{x} + \underbrace{r\sin(\omega t)}_{r_y}\hat{y}} \quad \text{—— (2)}$$

$|\vec{r}| =$

$$= \sqrt{(r_x)^2 + (r_y)^2}$$

$$= \sqrt{r^2\cos^2\omega t + r^2\sin^2\omega t}$$

$$= \sqrt{r^2\underbrace{(\cos^2\omega t + \sin^2\omega t)}_{=1}}$$

$$\boxed{|\vec{r}| = r}$$

the object moves on a circle. Its distance from the reference does not change in time.

Find $\left.\begin{array}{l}\vec{v}(t)\\ \vec{a}(t)\end{array}\right\}$?

$$\vec{r}(t) = r\,(\cos\omega t\ \hat{x} + \sin\omega t\ \hat{y})$$

$$\vec{v}(t) = \frac{d\vec{r}}{dt} = r\omega\left[-\sin\omega t\ \hat{x} + \cos\omega t\ \hat{y}\right] \qquad (3)$$

$$\vec{v}(t)\cdot\vec{r}(t)$$

$$= (v_x\hat{x} + v_y\hat{y})\cdot(r_x\hat{x} + r_y\hat{y})$$

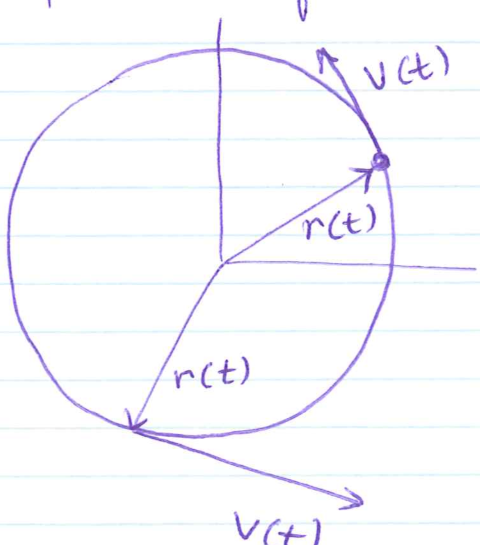
$$= v_x r_x + v_y r_y$$

$$= (-r\omega\sin\omega t)(r\cos\omega t) + (r\omega\cos\omega t)(r\sin\omega t)$$

$$= +r^2\omega\big[\underbrace{-\sin\omega t\cos\omega t + \sin\omega t\cos\omega t}_{=0}\big]$$

$$\boxed{\vec{v}\cdot\vec{r} = 0} \quad \text{at each time } t \qquad (4)$$

$\Rightarrow$ the velocity of the particle is tangent to the position of the particle at every time t



$$\underline{\vec{v} \perp \vec{r}} \quad \forall t$$

(5)

$$\boxed{|\vec{v}(t)| = r\omega}$$

Once again the speed of the particle is constant in time

$$\vec{v}(t) = r\omega\left[-\sin\omega t\,\hat{x} + \cos\omega t\,\hat{y}\right]$$

$$\vec{a}(t) = \frac{d\vec{v}}{dt}$$

$$= r\omega^2\left[-\cos\omega t\,\hat{x} - \sin\omega t\,\hat{y}\right]$$

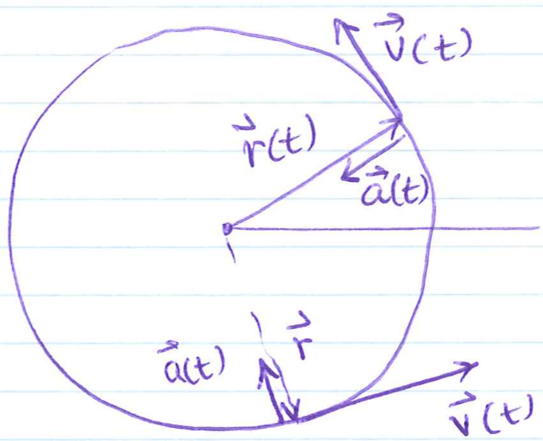
$$= -r\omega^2\left[\cos\omega t\,\hat{x} + \sin\omega t\,\hat{y}\right]$$

From Eq. (2)

$$\vec{r} = r\cos\omega t\,\hat{x} + r\sin\omega t\,\hat{y}$$

$$\Rightarrow \boxed{\vec{a}(t) = -\omega^2\,\vec{r}}$$

at every time t the acceleration of the particle is directed opposite to the radial vector $\vec{r}$.



$$|\vec{a}(t)| = -\omega^2 r$$

constant acceleration directed inwards at every time t.